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A MODEL FOR EVALUATING THE EFFECTIVENESS
OF A SYSTEMATIC SEARCH
FOR A MOVING TARGET

by

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The approach to the problem in this project is a modification of the methods of B.O. Koopman in *Search and Screening*. The method first calculates the target location density ρ at each point in a specified area A, and then integrates $(1-\rho)$ over A. The result is a measure of how effectively area A was searched.

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A Model for Evaluating the Effectiveness of a Systematic Search for a Moving Target

by

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ABSTRACT

The primary objective of this project is to estimate the effectiveness of a systematic search conducted against a randomly moving target and to generate target density curves in the search area after the search.

In this problem, searcher course and speed are fixed known values. The target course is also fixed, but is randomly selected. The measure of effectiveness (MOE) is the total equivalent area searched.

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I. INTRODUCTION

A. THE PROBLEM

The problem considered here is the search for a randomly moving target with a fixed speed and direction. We assume that the initial target distribution is uniform over a very large search area, and the searcher, carrying a cookie cutter sensor, is conducting a systematic search within that area. The question addressed here is, what is the expected area swept, $A_s(t)$, after t hours of search. We need to express our answer in terms of known parameters of target speed (u), searcher speed (v), sensor radius (R), and search time (t).

A standard approach to this problem is to assume that the expected relative speed between a searcher at speed v and a target at speed u is

$$w = (1/2\pi)\int_0^{2\pi}\sqrt{(u^2 + v^2 - 2uv\cos\theta)}\, d\theta.$$

Then the expected area searched by time t is $A_s(t) = 2Rwt$, where R is the detection range of the searcher. This model was introduced by Koopman in *Search and Screening* [Ref. 1]. In this thesis we will integrate $(1-\rho)$ over the area to calculate the equivalent area searched (A_s), where ρ is the posterior target density for any point in the area. The method is to derive analytic expressions for the equivalent area swept (when possible), to generate target density curves for the area, and then simulate some of the main results.

During this study, the following assumptions hold for each target at any time t.

1. Initial target position is uniformly distributed over the ocean.
2. Target course is uniformly disributed over $(0,2\pi)$.
3. Target position and course are independent random variables.

The model used in this project gives results agreeing with Koopman's model in stationary searcher and stationary target cases. In addition, when both target and searcher are moving with fixed speeds, the model developed here shows that the equivalent area swept is, contrary to intuition, the same for all $v \leq u$ in a specified area centered about the searcher. Moreover, the specific posterior target density profile is computed and may be of use in planning operations such as follow-on searches.

II. CALCULATION OF TOTAL DEPLETED AREA

A. THE MODEL

When a searcher is progressing on its course at the constant velocity v among a uniform random distribution of targets of speed u, it is frequently important to know the probability of a particular target passing within the stated range of R miles of the observer and hence being detected. In some cases R may be the range within which the searcher can sight the target. In others, it might be the range within which the target can detect the searcher's presence. Again R may be effective gun fire range of the searcher against the target, or vice versa. The problem we consider is this : Given a target located at (r,β), what is the probability $P(r,\beta)$ that the target in the past came within range R of the searcher?

Evidently P depends on (r,β) and $P=1$ if $r \leq R$. When $r \geq R$, the target has entered the circle if and only if its relative velocity vector w points to the detection circle. The angular range of the target course giving a detection sometime in the past is drawn in Fig. 2.1 (shaded angle). Then, given the assumption that target course is uniformly distributed between 0 and 2π radians, $P(r,\beta)$ becomes the ratio of the angular range of u to 2π. The problem is thus reduced to the geometry of Fig. 2.1, and the formula for $P(r,\beta)$ is obtained by straightforward trigonometry.

The expression of $P(r,\beta)$ is as follows: $P(r,\beta)=\Theta(r,\beta)/2\pi$ where $\Theta(r,\beta)$ is the total range in radians of u. This method was used by Koopman in *Search and Screening* [Ref. 1] Koopman calculated the probability $P(r,\theta)$ that the target at (r, β) will in the future come within range R of the searcher. The geometry in Koopman's problem is very similar to that studied here. In both cases a detection occurs whenever an extension of the target relative velocity vector comes within range R of the searcher. In Koopman's case, the relative velocity vector is extended forward to intersect the detection disk. Here it is extended backwards.

For later comparison with our more general result we will first look at the two extreme cases in which either the target or the searcher is stationary. We will be assuming a prior target density, ρ, of 1 target per unit area.

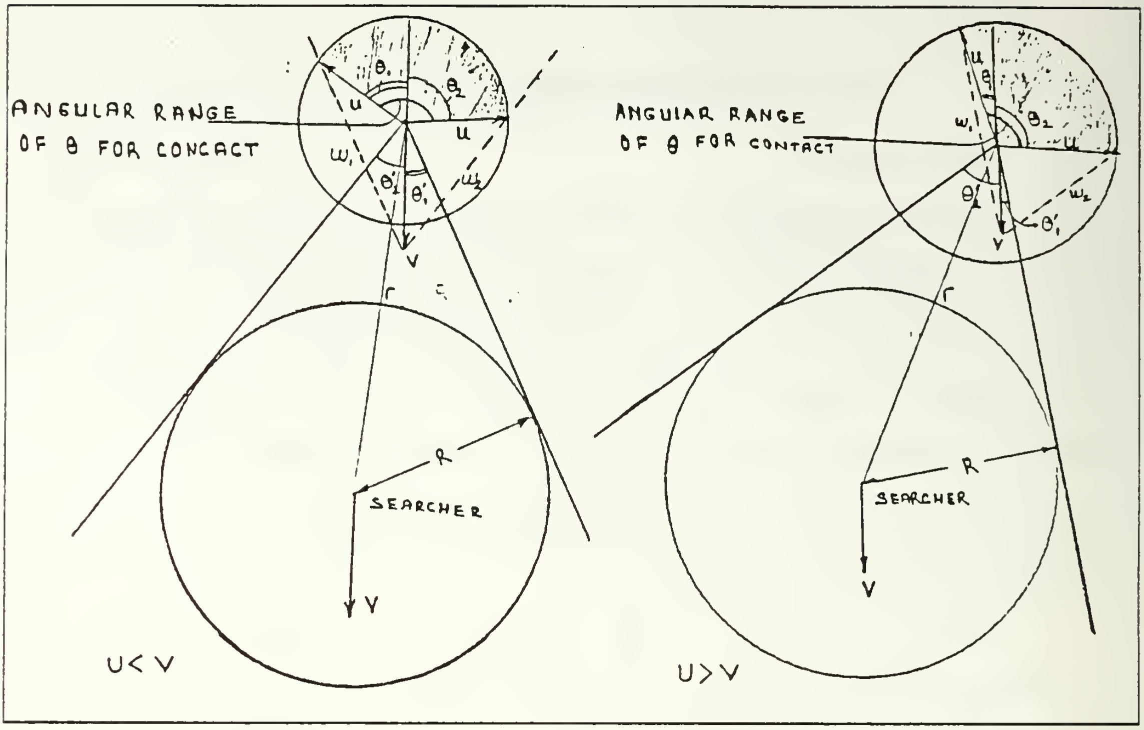

Figure 2.1 The Probability of Entering Circle.

B. STATIONARY TARGET

As seen in Fig. 2.2, if the target is fixed, the searcher develops a rectangular *"target-free tail"* of area 2Rvt, where t is search time. In this case target density ρ is 0 inside the target free tail and on the disk, and it is 1 outside these areas. The total swept area at time t, $A_t(t)$, is $\pi R^2 + 2Rvt$.

C. STATIONARY SEARCHER

Now let us consider the case in which the searcher is fixed. For a point at the distance $r \le ut+R$ from the center of the detection disk, the posterior target density is reduced by an amount proportional to the shaded angle in Fig. 2.1 Hence,

$$\rho(r) = 1 - \frac{\theta(r)}{2\pi}$$

Now simple geometry yields,

$$\rho(r) = \begin{cases} 1 - \dfrac{\arcsin(R/r)}{\pi} , & r \le \sqrt{(ut)^2 + R^2} \\ 1 , & r \ge ut + R, \end{cases}$$

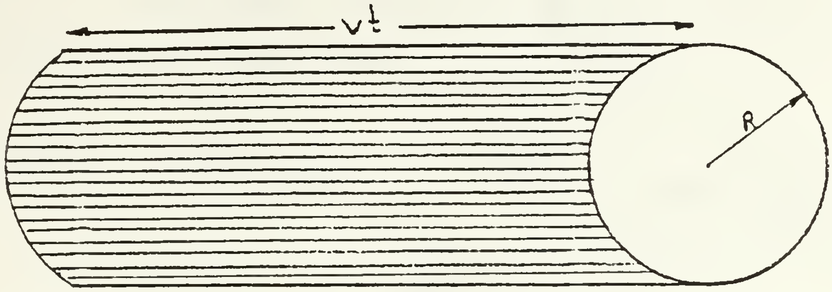

Figure 2.2 Searcher Motion In Stationary Target Area.

Note that we have not given an expression for $\rho(r)$ when r is in the interval $(\sqrt{(ut)^2+R^2}\ ,\ ut+R)$. Here the detection disk "edge effects" determine $\rho(r)$, as suggested in Fig. 2.3. For mathematical convenience we will generally ignore these edge effects and use the arcsine formula above to calculate $\rho(r)$ for all $r<(ut+r)$. The edge effect will be small for $ut>>R$. Note that $\rho(r)$ is 1 for all $r>ut+R$. This is because no targets can be captured by time t that start at a greater distance than $R+ut$. Neglecting the edge effect we have,

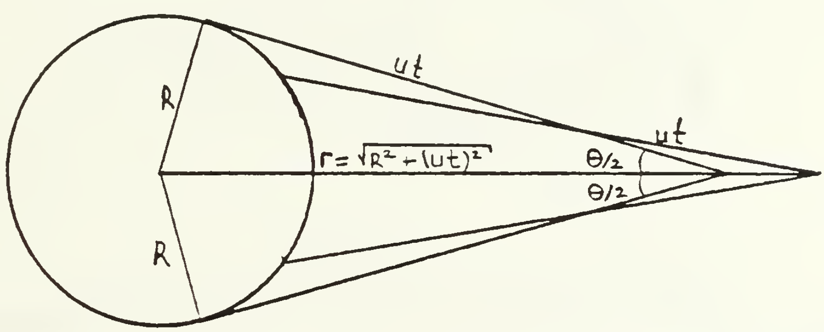

Figure 2.3 .

$$A_s(t) = \int_0^{2\pi}\int_R^{R+ut}(1-\rho(r))\, r\, dr\, d\theta + \pi R^2$$

$$A_s(t) = (1/\pi)\int_0^{2\pi}\int_R^{R+ut}\arcsin(R/r)\, r\, dr\, d\theta + \pi R^2$$

$$= 2\int_R^{R+ut}\arcsin(R/r)\, r\, dr + \pi R^2 \qquad \text{(eqn 2.1)}$$

After integrating the Equ. 2.1 (see [Ref. 2]) we substitute $\arcsin(R/r) \sim R/r$ (good when $r >> R$) and get

$$A_s(t) = uRt\{1+\sqrt{(1+2R/ut)}\} + R^2(1+\pi/2)$$

Eqn 2.1 overestimates the actual swept area because of the above mentioned edge effects on the detection disk. A lower bound can be found by evaluating the integral in Eqn. 2.1 only between R and $\sqrt{R^2+(ut)^2}$. Fig. 2.4 shows a plot of these upper and lower bounds, together with a plot of $2Rut+\pi R^2$ which, according to Koopman's result mentioned in Chapter I, should be the actual searched area by time t.

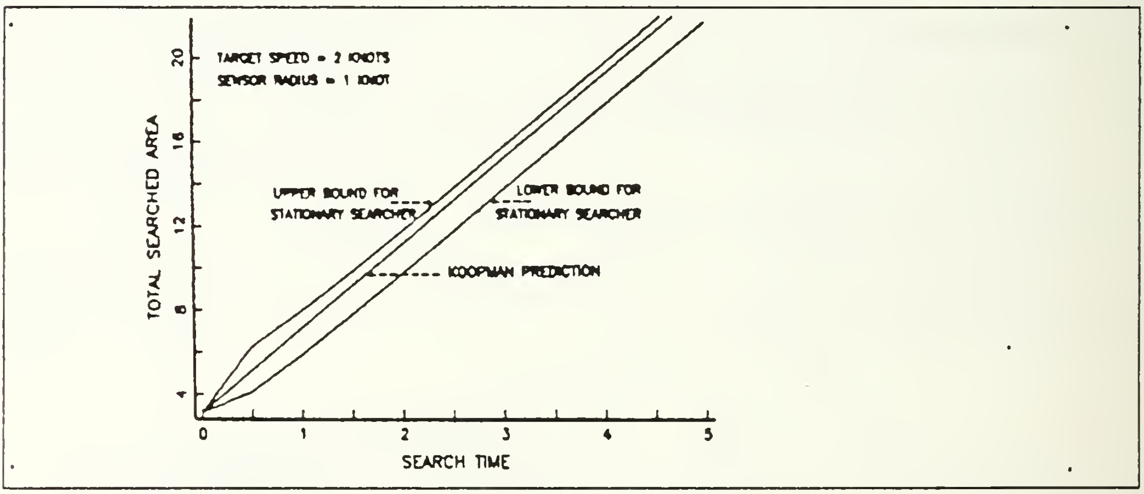

Figure 2.4 .

It is interesting to take the derivative of Eqn. 2.1 with respect to t to calculate the sweep rate.

$$\frac{dA_s(t)}{dt} = 2\,u\,(R+ut)\arcsin(R/(R+ut)),$$

which for small R and large t becomes

$$\frac{dA_s(t)}{dt} \sim 2u\,(ut+r)\frac{R}{(ut+r)} = 2uR$$

Hence,

$$A_s(t) = 2\,u\,R\,t \qquad \text{(eqn 2.2)}$$

We note that using this methodology when either the searcher or target is stationary is not particularly recommended, but is presented here to show that the methodology gives reasonable results in these simple cases and to demonstrate the method for a case in which the correct answer is known.

D. TARGET AND SEARCHER BOTH MOVING

In this problem both the target and the searcher are moving (Fig. 2.5). θ is the angle between the horizontal line and the target velocity vector u (corresponding to θ_1 or θ_2 in Fig. 2.1).

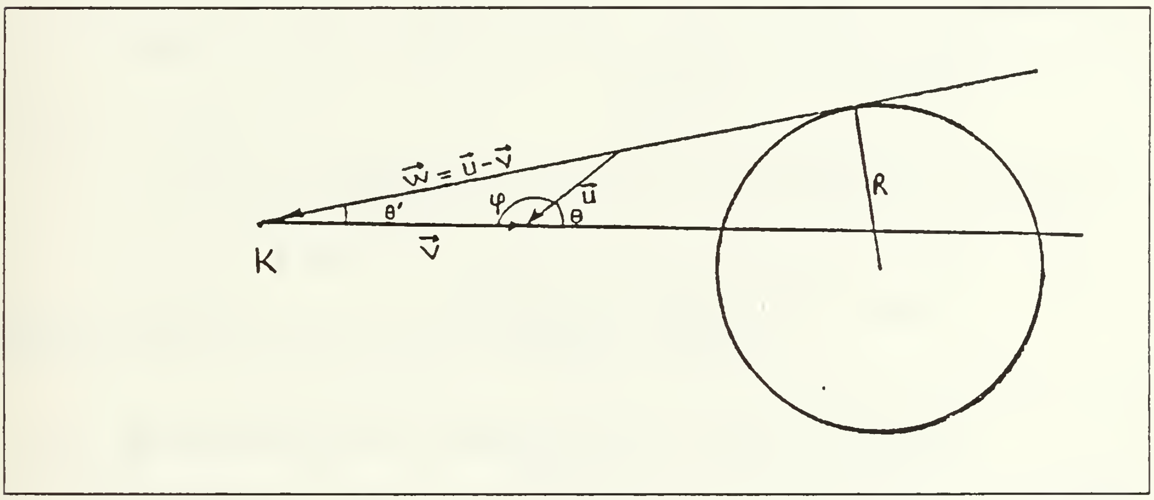

Figure 2.5 Relative Velocity Vector Diagram.

In the frame moving along with the searcher, targets starting from any given point can move in any arbitrary direction, but only the ones with velocity vector u inside of the shaded area in Fig. 2.1 could have been captured. So we need to find θ',

in the moving frame, and transform it to θ, in the stationary frame, to find the angle shielded by the disk for each point in the area. In Fig. 2.5,

$$-W\cdot V = wv\cos\theta' \qquad \text{(eqn 2.3)}$$

If $W = U - V$ is substituted in Equ. 2.3 we get

$$-(U-V)\cdot V = wv\cos\theta'$$

$$V\cdot V - U\cdot V = wv\cos\theta' \qquad \text{(eqn 2.4)}$$

Since $V\cdot V = v^2$ and $U\cdot V = -uv\cos\theta$, substituting into Equ. 2.4 yields

$$v^2 + uv\cos\theta = wv\cos\theta'$$

$$\cos\theta' = \frac{(v + u\cos\theta)}{w} \qquad \text{(eqn 2.5)}$$

By using the Cosine Law we get

$$w^2 = u^2 + v^2 - 2uv\cos\varphi$$
$$= u^2 + v^2 + 2uv\cos\theta$$

Now squaring both sides of the Eqn. 2.5 and substituting for w^2,

$$\cos^2\theta' = \frac{(v^2 + u^2\cos^2\theta + 2uv\cos\theta)}{(u^2 + v^2 + uv\cos\theta)} \qquad \text{(eqn 2.6)}$$

$$u^2\cos^2\theta' + v^2\cos^2\theta' + 2uv\cos\theta\cos^2\theta' - v^2 - u^2\cos^2\theta - 2uv\cos\theta = 0$$

$$u^2\cos^2\theta + 2uv\sin^2\theta'\cos\theta - (u^2 + v^2)\cos^2\theta' + v^2 = 0$$

The solution of this quadratic equation gives,

$$\cos\theta_{1,2} = \{-uv\sin^2\theta' \pm \sqrt{(uv\sin^2\theta')^2 + u^2(\cos^2\theta'(u^2+v^2) - v^2)}\} / u^2 \qquad \text{(eqn 2.7)}$$

So when we have θ' in the moving frame, we can transform it back to θ, in stationary frame. θ' can be found by trigonometry. For the following calculation let us take L as the horizontal axis, X is the vertical axis and the bottom of the detection circle is the coordinate center (Fig. 2.6). Using the notation in Fig 2.6,

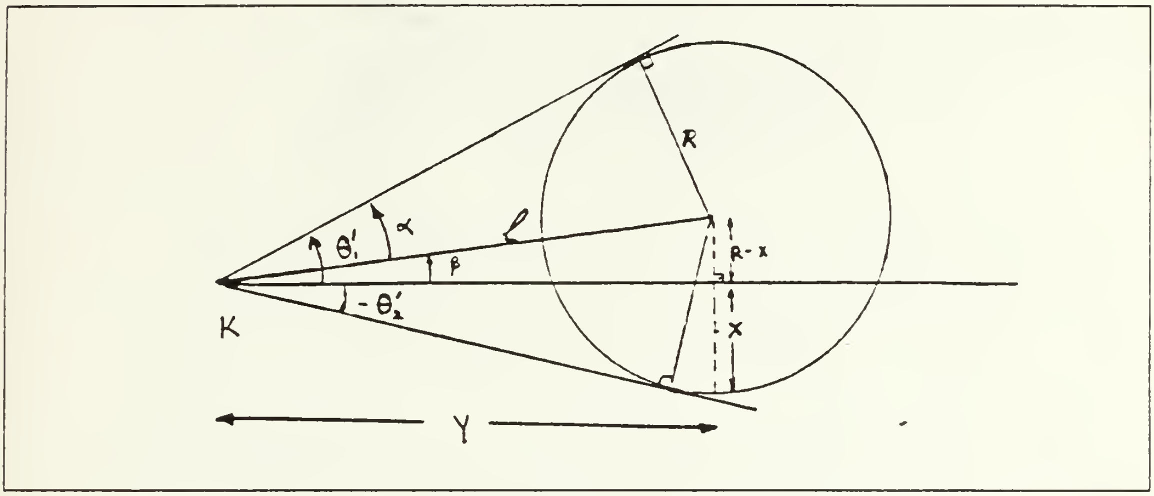

Figure 2.6 .

$\sin\alpha = R / l$ and $\tan\beta = R\text{-}x / Y$.

Since $l^2 = (R\text{-}x)^2 + Y^2$,

$\alpha = \arcsin\{r / \sqrt{(R\text{-}x)^2 + Y^2}\}$ and $\beta = \arctan\{(R\text{-}x)/Y\}$

$\theta_1' = \alpha + \beta$ and $-\theta_2' = \alpha - \beta$

By subtituting the α and β values in this formula we get

$$\theta_1' = \alpha + \beta = \arcsin\{R / \sqrt{(R\text{-}x)^2 + Y^2}\} + \arctan\{|R\text{-}x| / Y\}$$

$$-\theta_2' = \alpha - \beta = \arcsin\{R / \sqrt{(R\text{-}x)^2 + Y^2}\} - \arctan\{|R\text{-}x| / Y\} \qquad \text{(eqn 2.8)}$$

So we have calculated $\theta'_{1,2}$. We can now transform these angles to $\theta_{1,2}$ by using our transformation formula, Eqn. 2.7. The shielded angle for any given point in the area will be $|\theta_1 - \theta_2|$, and target density for that point is $\rho = \{1 - |\theta_1 - \theta_2| / 2\pi\}$.

Earlier, we selected a point on one side of the geometric tail and derived Equ. 2.8. For symmetric points on the opposite side of the tail, θ'_1 and θ'_2 do not change (Fig. 2.7) and Eqn. 2.8 still holds.

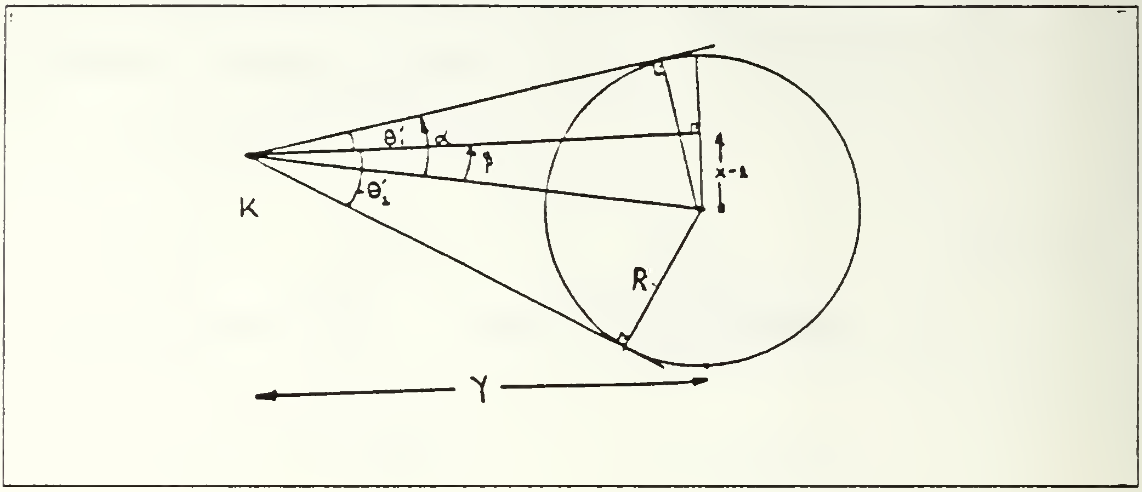

Figure 2.7 .

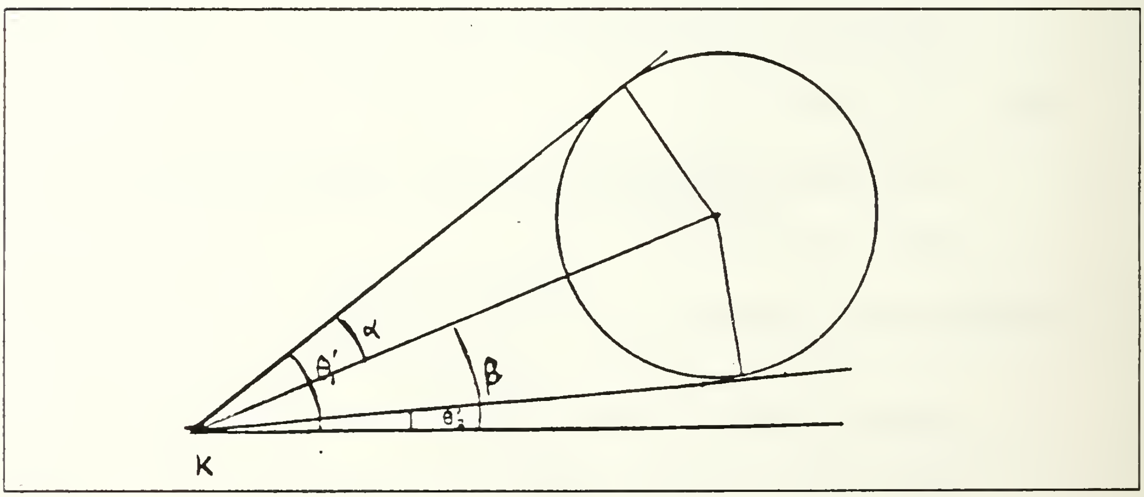

Figure 2.8 .

If we consider the same problem for the region outside of the geometric tail (Fig. 2.8) and derive an analytic expression for θ'_1 and θ'_2 we have,

$$\alpha = \arcsin\left[R / \sqrt{(R-x)^2 + Y^2}\right] \quad \text{and} \quad \beta = \arctan\left[(R-x) / Y\right]$$

$$\theta_1' = \alpha + \beta = \arcsin[R / \sqrt{(R\text{-}x)^2 + Y^2}] + \arctan[(R\text{-}x) / Y]$$

$$\theta_1' = \alpha - \beta = \arcsin[R / \sqrt{(R\text{-}x)^2 + Y^2}] - \arctan[(R\text{-}x) / Y]$$

These are the same expressions obtained in Equ. 2.8. So these equations can be used to find the density $\rho = \{1\text{-}|\theta_2 - \theta_1| / 2\pi\}$ for any given point.

We cannot find the total effective area searched (A_s) by closed form integration as we did before when the searcher was fixed. Instead we find θ' and transform it to θ, then calculate total effective area searched by numerical integration. Let $k = v/u$ for computational purposes.

We can use the computer programs given in Appendix B to calculate densities for each point in the area considered. Plotting of these values gives density curves in Fig. 2.9 for the case in which target is moving twice as fast as the searcher. Fig. 2.9 and 2.11 show that density curves propagate like a sound or shock wave in the air. Note that the horizontal and vertical scales are different in these figures. The contours in Fig. 2.9 are actually circles. In Fig. 2.11, if we look closely at the forward-most contours and the target free tail (the area in which $\rho = 0$), we can see that they follow the Cherenkov formula [Ref. 3] for shock front directions. We will see the shape of the target free tail in more detail in Appendix A.

When $k = 2$, the searcher is moving twice as fast as the target, and the searcher develops an empty tail behind. This target free tail starts to develop for $k > 1$ and gets larger and larger as k increases as is seen in Fig. 2.13.

It is important to note that the target densities calculated are steady state values. That is, they would result only after the search has been in progress for a sufficiently long time. A "sufficiently long time" for any point x is the maximum time required for a target starting on the edge of the detection disk to reach point x in searcher-stationary relative space.

After running the programs in Appendix B for $u = 2$ and $v = 1$ and plotting them, we obtain the graphs given in Fig. 2.9 and 2.10. Now the question is what is the total swept area A_s? The total swept area after search for a time t (where $ut >> R$, so we can neglect edge effects) is obtained by integrating $(1\text{-}\rho)$ over the entire search area. Since this is not possible to do analytically, we use a numerical integration method. According to Koopman's model, this area will be proportional to w. Our results and the simulation presented in Appendix C are consistent with this.

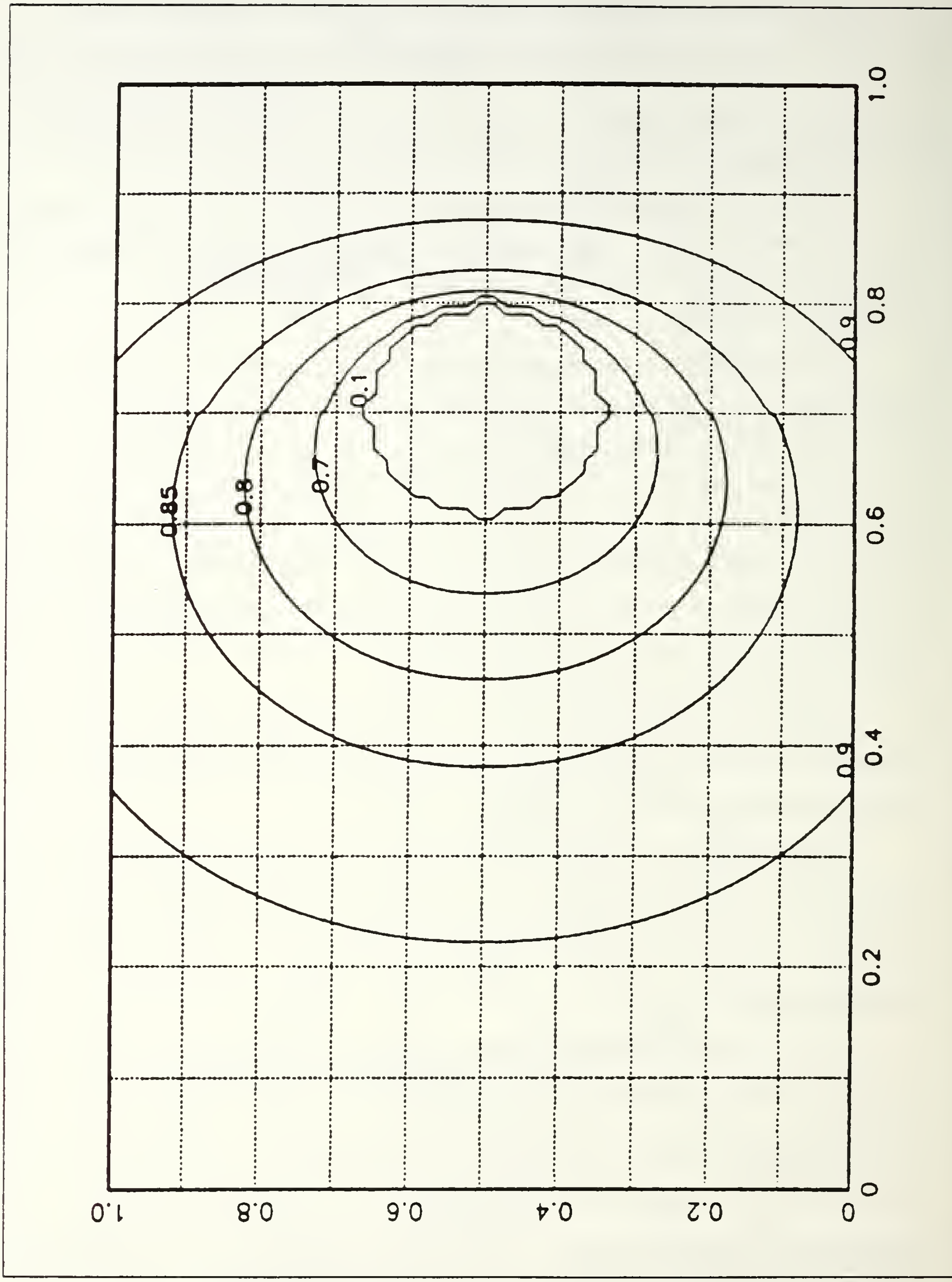

Figure 2.9 Density Curves k=.5.

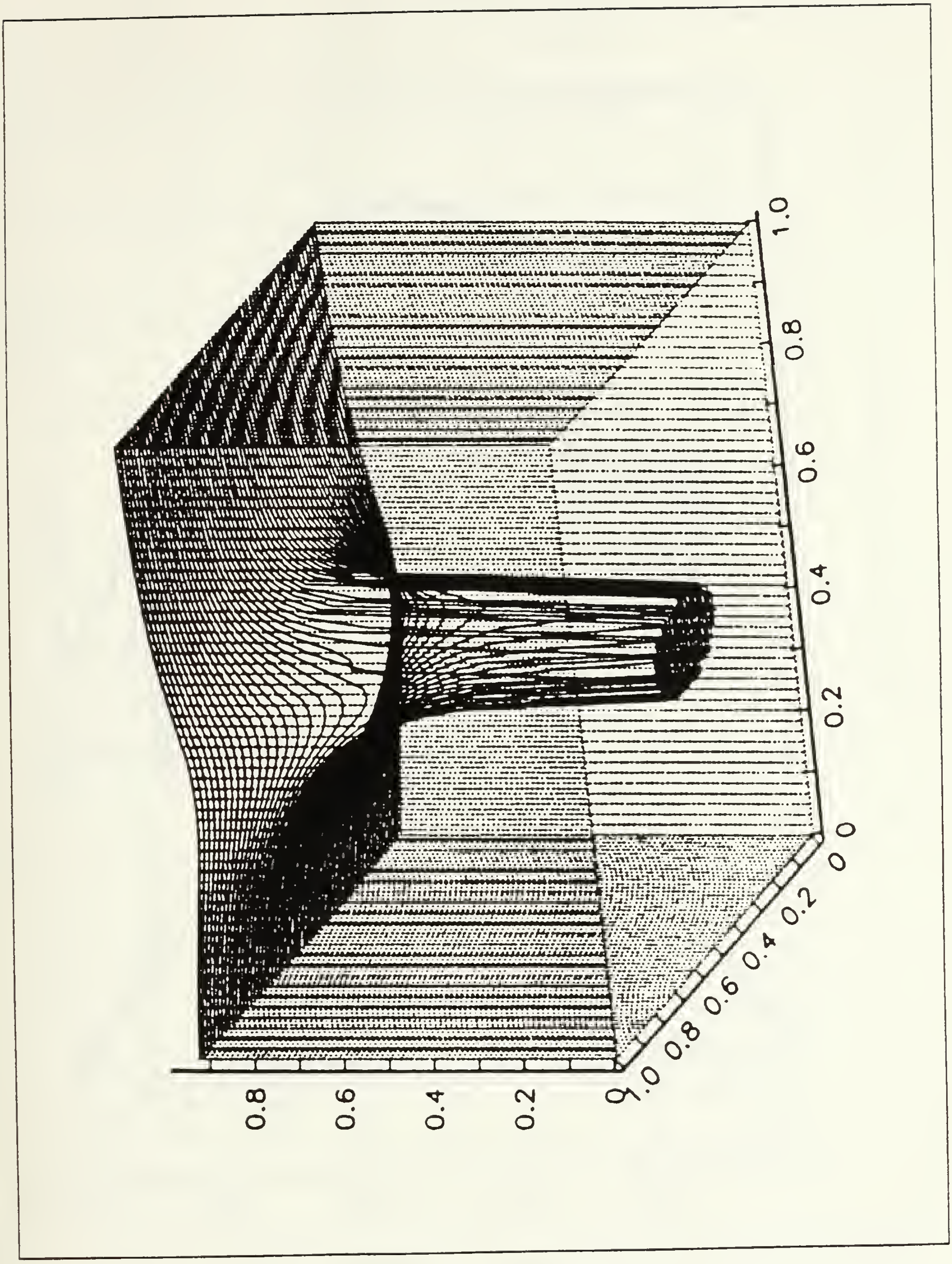

Figure 2.10 Density Surface k = .5.

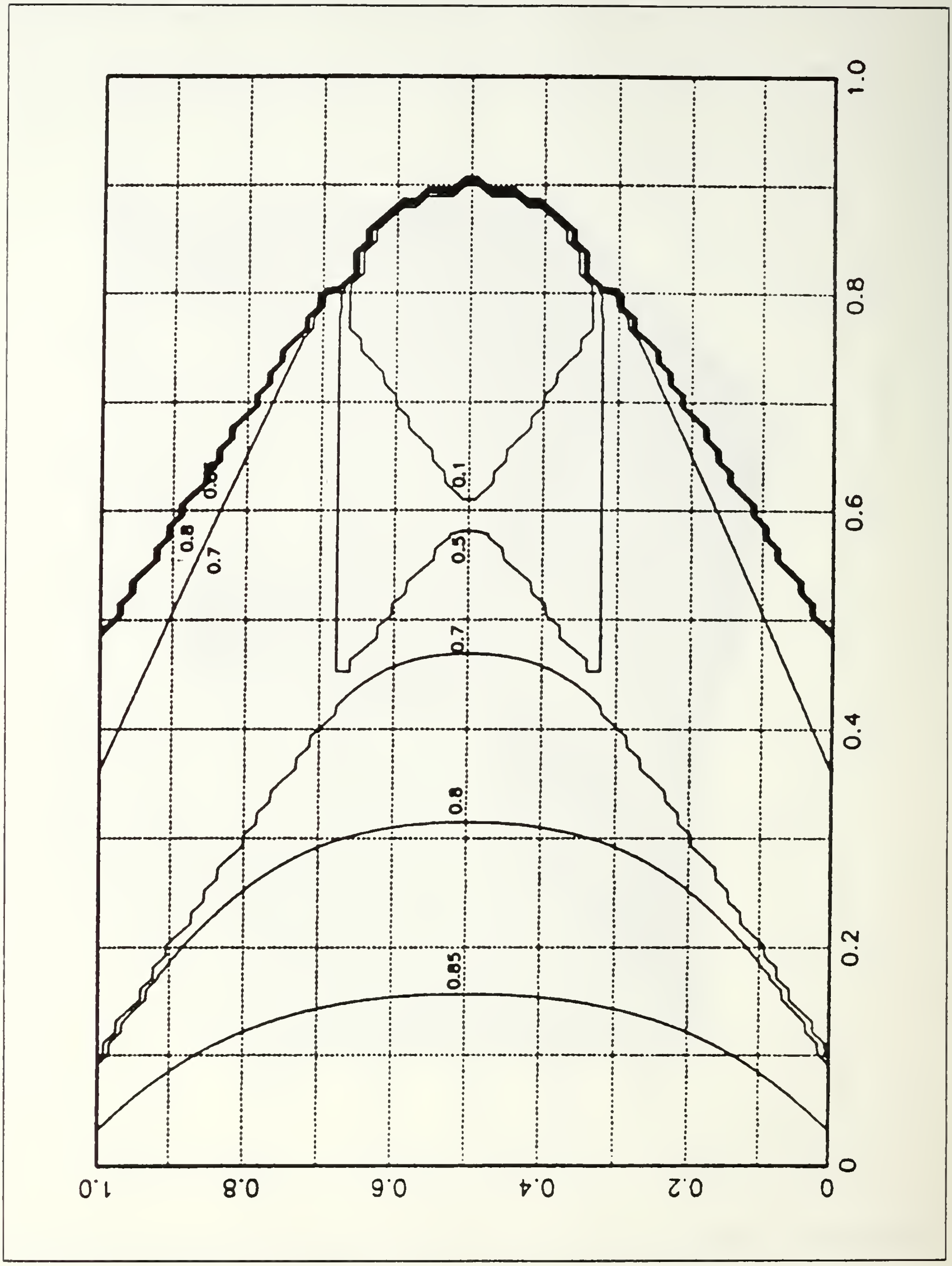

Figure 2.11 Density Curves k = 2.

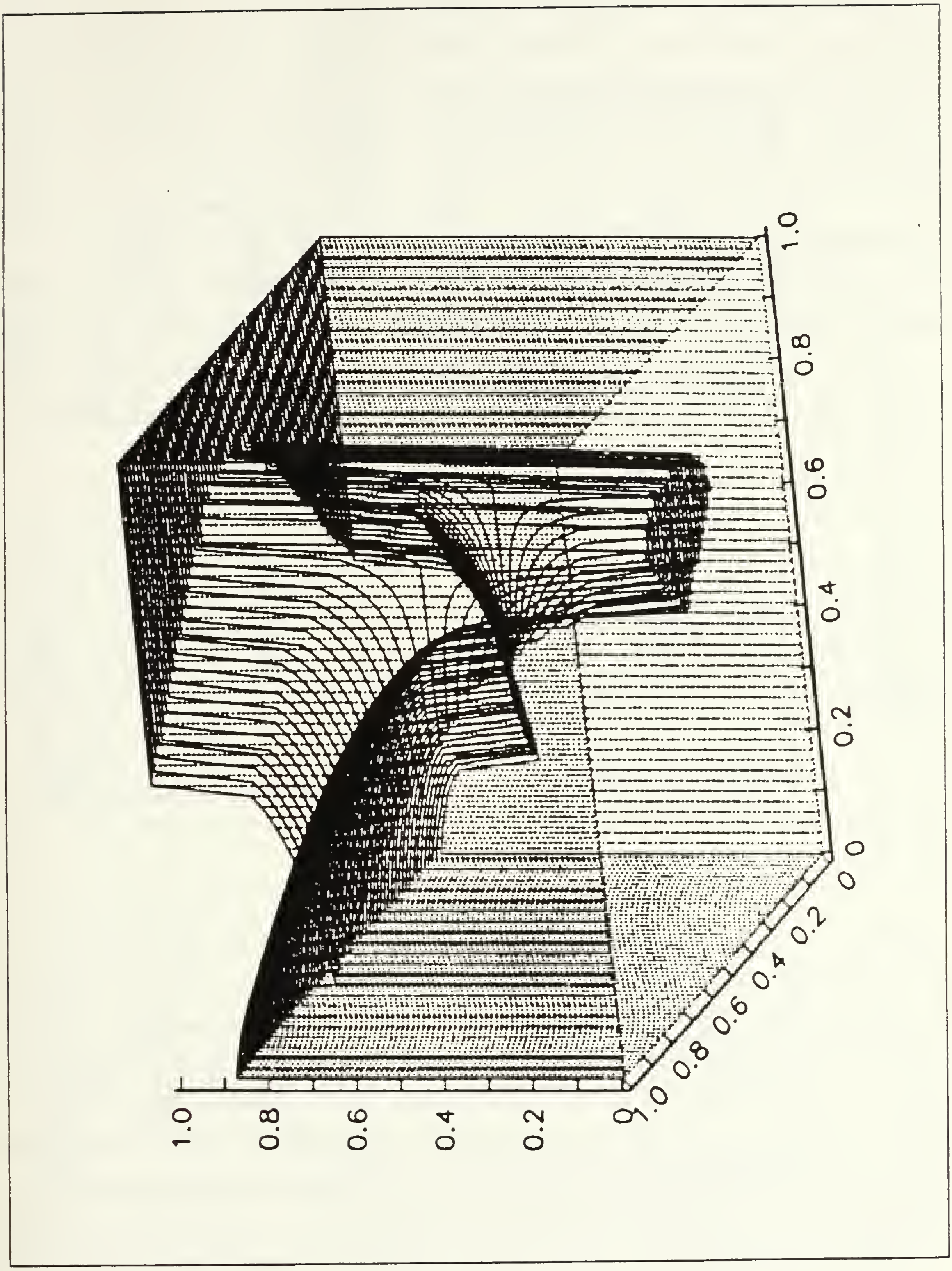

Figure 2.12 Density Surface k = 2.

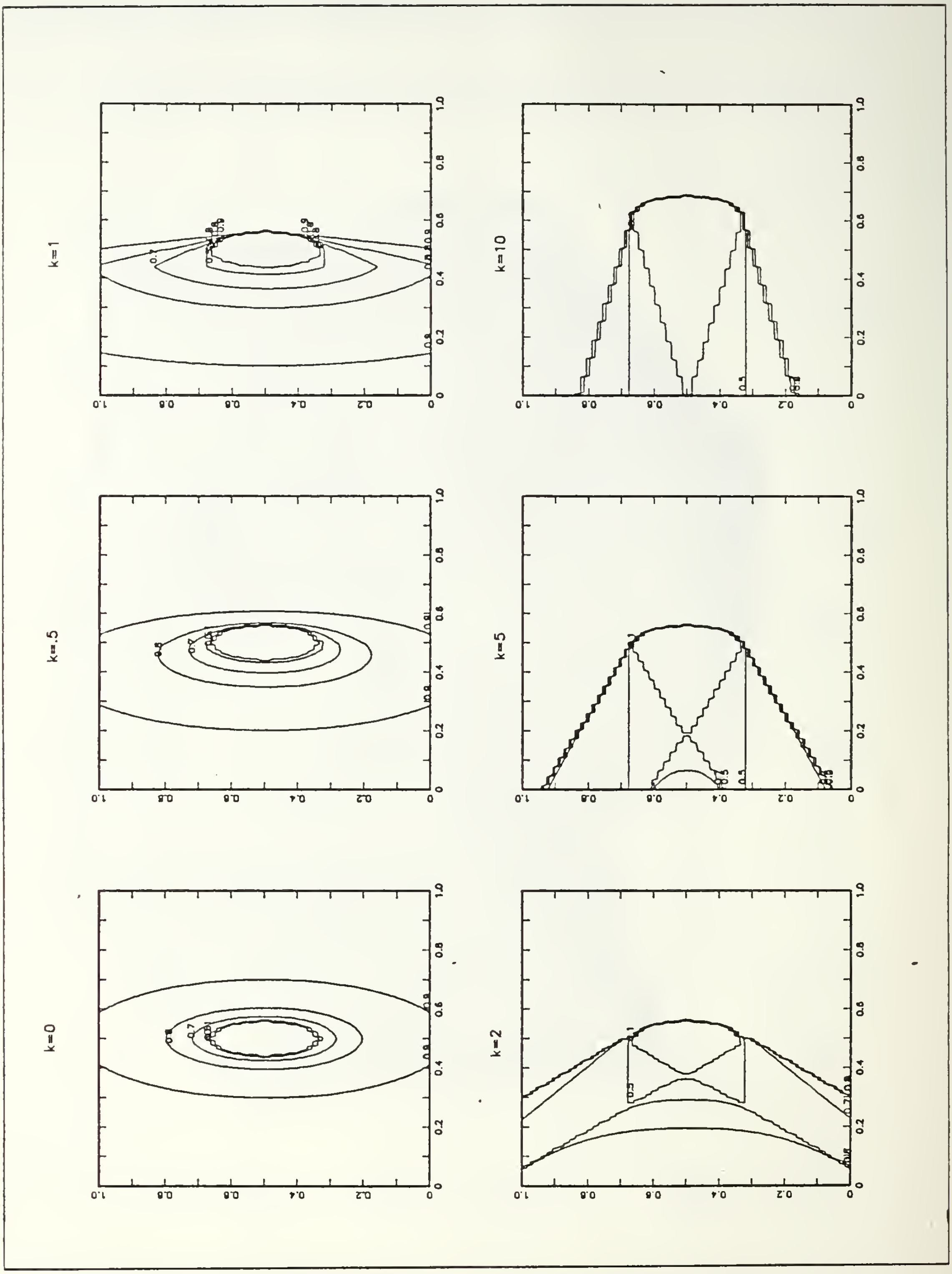

Figure 2.13   Development of the Tail.

However, we have found that integrating 1-ρ over any finite area centered about the present searcher position yields a constant for any $v \leq u$, for a given u. This results because the amount of increase in density in front of the searcher is the same as the amount of decrease in density behind the searcher for a fixed target speed $u \geq v$. That results in the same total effective area searched whenever the relevant area A has a front-to-back symmetry about the searcher.

To see why this can be so, in Figure 2.14 select one point in front of the searcher (K′) and another point behind the searcher (K) which is symmetric with respect to the searcher. Now take a close look at these points to see what is happening as v changes and u remains fixed.

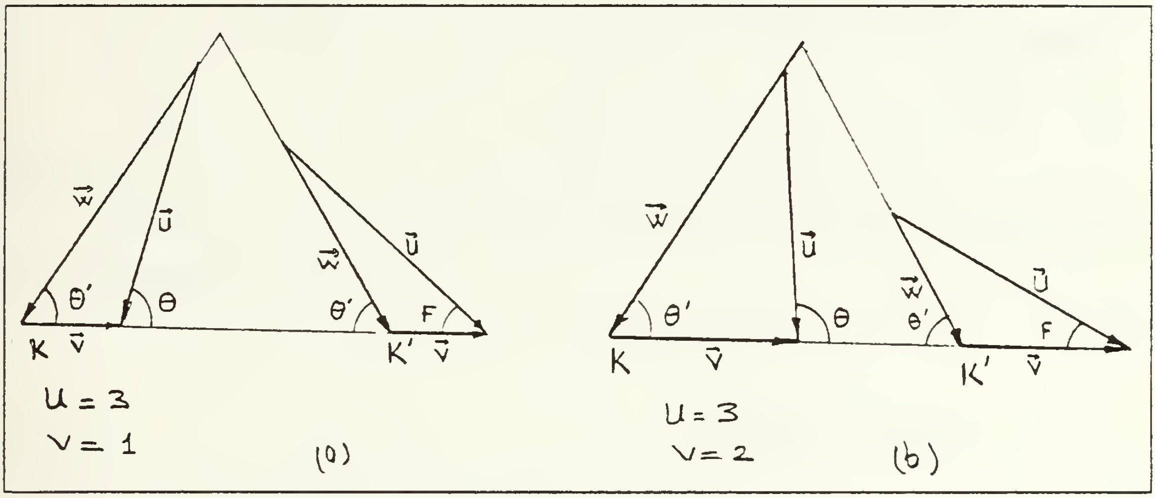

Figure 2.14 .

In Fig. 2.14a we draw a velocity vector diagram for v = 1 and u = 3 to see θ and θ′ angles which we had calculated by computer program. θ and F represent the shielded sector by the searcher. Now let us keep u = 3 and change v from 1 to 2. As seen in Fig. 2.14b, as v increases, θ increases but F decreases. In order for (F + θ) to remain constant, the amount of increase behind the searcher must be equal to the amount of decrease in front of the searcher. Hence, the increase in ρ at the point behind the searcher is compensated by a decrease in ρ at the point ahead of the searcher. This gives us an intuitive idea about what is occuring as v increases and u is kept fixed, but we still need to prove it.

In Section II.d we picked a point K (Fig. 2.14) behind the searcher and derived a formula for $\cos\theta$ (eqn 2.7). By using that formula and letting k = v u we get

$$\theta_{1,2} = \arccos\{ k[-\sin^2\theta' \pm \cos\theta' \sqrt{(k^2-\sin^2\theta')}] \} \qquad \text{(eqn 2.9)}$$

Now let us apply the same method to the point K′ in front of the searcher and derive an expression for F. In Fig. 2.14,

$$W = U-V$$

$$-W\cdot V = wv\cos(\pi - \theta'), \text{ so}$$

$$W\cdot V = wv\cos\theta$$

Also,

$$W\cdot V = (U-V)\cdot V = U\cdot V-V\cdot V \qquad \text{(eqn 2.10)}$$

Substitute $V\cdot V = v^2$ And $U\cdot V = uv\cos F$ in eqn. 2.10 to yield

$$W\cdot V = wv\cos\theta' = uv\cos F - v^2$$

$$\cos\theta' = \frac{u\cos F-v}{w} \qquad \text{(eqn 2.11)}$$

By using cosine theorem $w^2 = u^2 + v^2 -2uv\cos F$

$$\cos^2\theta' = \frac{u^2\cos^2F+v^2-2uv\cos F}{v^2+u^2-2uv\cos F}$$

Solving this equation for cosF gives

$$\cos F_{1,2} = k[\sin^2\theta' \pm \cos\theta\sqrt{(k^2-\sin^2\theta')}]$$

$$F_{12} = \arccos\{ k[\sin^2\theta' \pm \cos\theta \sqrt{(k^2-\sin^2\theta')}] \}.$$

If we look at the argument inside of the { } in this expression, we notice that it is the negative of the argument inside of the { } in eqn 2.9.

Now applying the chain law,

$$
\begin{aligned}
\frac{d\theta_{1,2}}{dv} &= \frac{d}{dx}\arccos(x)\,\frac{d}{dv}\left\{v/u\left[-\sin^2\theta' \pm \cos\theta'\sqrt{(v/u)^2-\sin^2\theta'}\,\right]\right\} \\
&= -\frac{d}{dx}\arccos(x)\,\frac{d}{dv}\left\{v/u\left[\sin^2\theta' \pm \cos\theta'\sqrt{(v/u)^2-\sin^2\theta'}\,\right]\right\} \\
&= -\frac{dF_{1,2}}{dv}
\end{aligned}
$$

That means, as searcher speed increases, the amount of increase of shielded angle behind the target is equal to the amount of decrease of shielded angle in front of the searcher. Since the amount of change of shielded angle at each point will be compensated by symmetrical points with respect to the searcher and swept area is equal to the sum of shielded angles over the area, there will be no change in equivalent area swept for a symmetrical area around the searcher's present position. This result does *not* mean that we will get the same total equivalent swept area for different $v \leq u$ values at a particular time t. The calculation of that area integrates $(1-\rho)$ over a circular area of radius $R+ut$ centered on the *initial* searcher position. If we consider the time dimension and the searchers initial position, the equivalent swept area will in general change with u, v and t.

III. CONCLUSION

In searching in an area, a major concern is to calculate the swept area by the time t. This information can be used to determine how effective the search is, the number of searchers to be allocated to this area and how long this search should be conducted. Koopman approached this problem with the expression for swept area, $A(t) = 2Rwt$, where w is the expected relative speed. In this project we examined the same model, with a different approach calculating the total swept area as a function of target and searcher parameters. The reason we used this model is that the area swept is not just that behind the searcher unless the target is stationary. We need to know how this search changes the initial target density at different points inside of the affected area.

Since we considered a very large ocean, we did not attempt to compute the percentage of the area swept. We defined the MOE as either the total area swept by the time t or equivalently, the number of detections during that time, assuming a uniform initial target density. For extreme cases (stationary target and stationary searcher) we found the total swept area analytically as a function of the relevant parameters obtaining the same total swept area as in the Koopman model (which calculates the total swept area by using expected relative speed). Suprisingly, we saw that when the target and the searcher are both moving, the amount of change in density in front of the searcher is the same as the amount of change in density behind the searcher, as v changes for a fixed $u \geq v$. For a search area, with front-to-back symmetry about the searcher's present psition, and after a sufficiently long search (defined in Chapter II), the total swept area is constant for any search speed v less than or equal to the target speed u.

The analytic result for total swept area by the time t when *both* the target and the searcher were moving was confirmed by use of the simulation program in Appendix C and is consistent with our detailed numerical evaluation of posterior target densities. Also the target density curves can be useful for determining optimal search patterns and efficient locations for the other searchers. Other extensions of our analysis include targets moving with an arbitrary distribution of speed. Through further analysis using an evaluation of posterior target densities this distribution of

target speed could determine the optimal search speed and search pattern. These applications have not been pursued here.

APPENDIX A
THE SHAPE AND THE AREA OF THE TARGET FREE TAIL

Let us examine the target free tail that the searcher develops behind. If we look at our transformation formula (eqn 2.7), we can see that this transformation will not work inside the target free tail. This is because any point inside this tail is shielded in all directions. In other words there is no way for any of the targets in the area to get in this tail. This happens only when the discriminant of equation 2.7 is negative. So if we make this discriminant equal to zero, we can determine the boundaries of the target free tail.

$$(uv\sin^4\theta')^2 + u^2(\cos^2\theta'(U^2 + v^2) - v^2) = 0$$

$$v^2 \sin^4\theta' - (u^2 + v^2)\sin^2\theta' + u^2 = 0$$

By solving this equation we get

$$\sin\theta_1{}' = u / v \quad \text{and} \quad \sin\theta_2{}' = -u / v$$

This the same formula used to calculate the angles at which shock waves propagate from supersonic aircraft. The generation of this waves can be seen in Fig. 2.9 and 2.11 for a few different values of $k = v/u$.

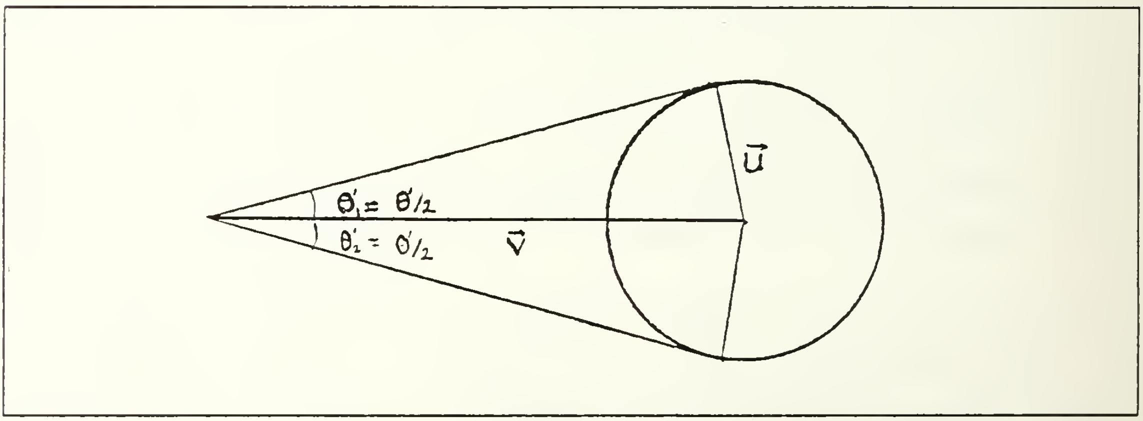

Figure A.1 .

So we can say that this tail only happens when $u < v$. It starts to develop when $u = v$, like a shock wave, which is what we see in Fig. 2.13.

Let r be the length of the tail. It is easily computed.

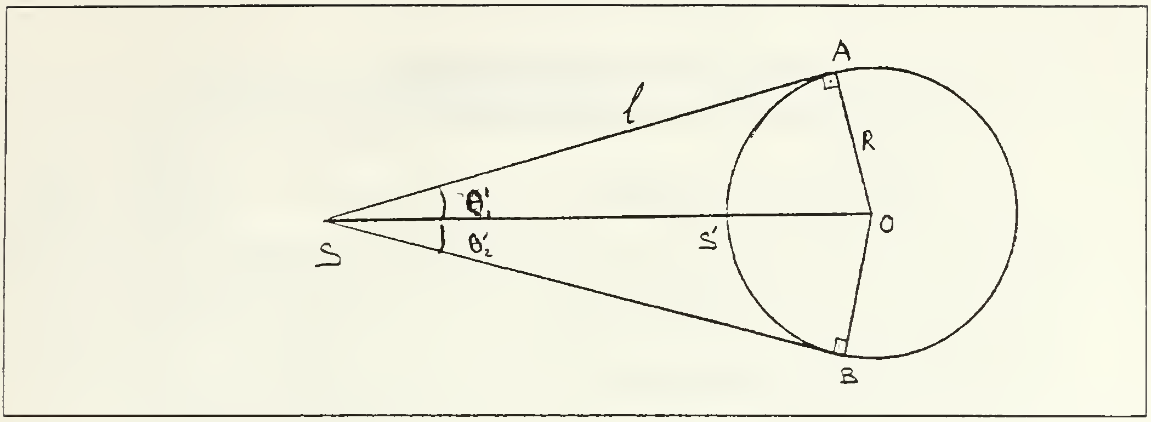

Figure A.2 .

We have that

$$\sin(\theta'/2) = u / v = R / r, \quad \text{so} \quad r = R(v/u).$$

Time (t') to develop this tail is $t' = r / v$.

The area of the tail will be, 2Area(ASO) - Area(AS'B) (Fig A.2). And

$$\begin{aligned} \text{Area(ASO)} &= (1/2)R\, l \\ &= (1/2)R\sqrt{(r^2 + R^2)} \\ &= (1/2)R\sqrt{R^2 + (R(v/u))^2} \\ &= (1/2)R^2\sqrt{1 + (v/u)^2} \end{aligned}$$

Area(AS'B) = αR^2 and α = arccos(R/r). After substitution we can find the area of the target free tail.

$$\begin{aligned} \text{Area} &= R^2\sqrt{1 + (v/u)^2} - R^2 \arccos(u/v) \\ &= R^2[\sqrt{1 + (v/u)^2} - \arccos(u/v)]. \end{aligned}$$

APPENDIX B
FUNCTIONS

All functions in this Appendix are used to calculate the shielded angle at any point in the area. In order to run these programs it is enough to enter target speed (u), searcher speed (v) and searcher radius (R) as a vector in function DENST. This function runs all the other necessary functions as a subfunctions. The output density matrix can be found in variable RH.

```
      ∇ DENST D;U;V;R;T1VV;T2VV;T1;T2
[1]  U←D[1]
[2]  V←D[2]
[3]  R←D[3]
[4]  ANGLE D
[5]  →(D[1]>D[2])/F1
[6]  →(D[1]<D[2])/F2
[7]  D[1]←D[2]+0.0001
[8]  F1:D TRANS1 T1VV
[9]  D TRANS2 T2VV
[10] T1 RRHO T2
[11] →100
[12] F2:D TRAN1 T1VV
[13] D TRAN2 T2VV
[14] T1 RHO T2
      ∇
```

Figure B.1 .

Function ANGLE uses target speed (u), searcher speed (v) and searcher radius (R) as input and generates T1VV (θ'_1) and T2VV (θ'_2).

TRANS1 TRANS2 and RRHO are used when the target is faster than the searcher. Function TRANS1 uses θ'_1(T1VV) as input and transforms it to θ_1 (T1).

Function TRANS2 uses θ'_2 (T2VV) as input and transforms it to θ_2 (T2). Function RRHO uses θ_1 (T1) and θ_2 (T2) as input to calculate densities ρ (RH) and puts zeroes for densities on the detection disk.

TRAN1 TRAN2 and RHO are used when the target is slower than the searcher. Function TRAN1 uses θ'_1(T1VV) as input and transforms it to θ_1 (T1).

Function TRAN2 uses θ'_2 (T2VV) as input and transforms it to θ_2 (T2).

```
∇ ANGLE D;U;V;R;M;ALFA;BETA;T1P;T2P;TV;T;P1;P2;P3
⍝    T1;T2;RHO;T1V;T2V;X;Y;
⍝U:TRGT VELOCITY/V:SRCHR VELOCITY/R:SRCHR RADIOUS
 RHO1←T1V←T2V←0
 U←D[1]
 V←D[2]
 R←D[3]
 X←0.999
L:Y←20×R
L1:M←(((X-R)*2)+(Y*2))*0.5
 →(M>R)/L4
 T1P←T2P←○0.5
 →L3
L4:ALFA←(¯1○(R÷((((R-X)*2)+Y*2)*0.5)))
 BETA←(¯3○((R-X)÷Y))
 T1P←BETA+ALFA
 T2P←BETA-ALFA
L3:T1V←T1V,T1P
 T2V←T2V,T2P
 →((Y←Y-0.1×R)≥0)/L1
 →((X←X-0.1×R)>¯2×R)/L
 T1V← 150 200 ⍴(1↓T1V)
 T2V← 150 200 ⍴(1↓T2V)
 T1VV←T1V,((○1)-⌽T2V)
 T2VV←T2V,((○1)-⌽T1V)
∇
```

Figure B.2 .

```
∇ D TRANS1 K;U;V;R;P1;P2;P3;TT1;TT2;T1D;T1U;T1UL;T1UR
⍝T1UML;T1UMR
⍝D:AS DEFINED IN DENST FUNCTION
⍝K:TETA1 PRIME ANGLE MATRIX WHICH IS 'T1VV' IN DENST FUNC.
 U←D[1]
 V←D[2]
 R←D[3]
 P1←¯2×U×V×((1○K)*2)
 P2←P1*2
 P2←(P2+4×(U*2)×((((2○K)*2)×((U*2)+(V*2)))-V*2))*0.5
 P3←2×(U*2)
 TT1←¯2○((P1+P2)÷P3)
 TT2←¯2○((P1-P2)÷P3)
 T1D←TT1[10+⍳140;⍳190],TT2[10+⍳20;190+⍳210]
 T1UL←TT1[⍳10;⍳190]
 T1UML←TT2[⍳10;190+⍳10]
 T1UR←(○2)-TT2[⍳10;200+⍳200]
 T1U←T1UL,T1UML,T1UR
 T1←T1U,[1] T1D
∇
```

Figure B.3 .

```
    ∇ D TRANS2 K;U;V;R;P1;P2;P3;TT1;TT2;T2D;T2UL;T2UR;T2U
  ⍝D:AS DEFINED IN DENST FUNCTION
  ⍝K:TETA2 PRIME ANGLE MATRIX WHICH IS 'T2VV' IN DENST FUNC.
  U←D[1]
  V←D[2]
  R←D[3]
  P1←¯2×U×V×((1○K)*2)
  P2←P1*2
  P2←(P2+4×(U*2)×((((2○K)*2)×((U*2)+(V*2)))-V*2))*0.5
  P3←2×(U*2)
  TT1←¯2○((P1+P2)÷P3)
  TT2←¯2○((P1-P2)÷P3)
  T2D←TT1[10+⍳140;⍳210],TT2[10+⍳140;210+⍳190]
  T2UL←¯1×TT1[⍳10;⍳200]
  T2UR←TT1[⍳10;200+⍳10],TT2[⍳10;210+⍳190]
  T2U←T2UL,T2UR
  T2←T2U,[1] T2D
    ∇
```

Figure B.4 .

```
    ∇ T1 RRHO T2;X;Y;XY
  RH←1-(T1-T2)÷○2
  RH←(⊖RH),[1] RH
  X←⍳200
  Y←⍳150
  XY←(((X*2)∘.+(Y*2))*0.5)>10.1
  XY←(⌽XY),XY
  XY←⍉(⊖XY),[1] XY
  RH←RH×XY
    ∇
```

Figure B.5 .

Function RHO uses θ_1 (T1) and θ_2 (T2) as input to calculate densities ρ (RH) and puts zeroes for densities on the detection disk.

```
     ∇ D TRAN1 K;U;V;R;P1;P2;P3;TT1;TT2;T1D;T1U;T1UL;T1UR
[1]  ⍝T1UML;T1UMR
[2]  ⍝D:AS DEFINED IN DENST FUNCTION
[3]  ⍝K:TETA1 PRIME ANGLE MATRIX WHICH IS 'T1VV' IN DENST FUNC.
[4]  U←D[1]
[5]  V←D[2]
[6]  R←D[3]
[7]  P1←¯2×U×V×((1○K)*2)
[8]  P2←P1*2
[9]  P2←(P2+4×(U*2)×((((2○K)*2)×((U*2)+(V*2)))-V*2))
[10]  I←P2≥0
[11]  P2←I×P2
[12]  P2←P2*0.5
[13]  P3←2×(U*2)
[14]  J←1≥|(P1+P2)÷P3
[15]  TT1←(○(1-J))+J×(¯2○(J×((P1+P2)÷P3)))
[16]  TT2←(○(1-J))+J×(¯2○(J×((P1-P2)÷P3)))
[17]  TT1←(○(1-I))+I×TT1
[18]  TT2←(○(1-I))+I×TT2
[19]  T1D←TT1[10+⍳140;⍳190],TT2[10+⍳140;190+⍳210]
[20]  T1UL←TT1[⍳10;⍳190]
[21]  T1UML←TT2[⍳10;190+⍳10]
[22]  T1UR←(○2)-TT2[⍳10;200+⍳200]
[23]  T1U←T1UL,T1UML,T1UR
[24]  T1←T1U,[1] T1D
     ∇
```

Figure B.6 .

```
     ∇ D TRAN2 K;U;V;R;P1;P2;P3;TT1;TT2;T2D;T2UL;T2UR;T2U
[1]  ⍝D:AS DEFINED IN DENST FUNCTION
[2]  ⍝K:TETA2 PRIME ANGLE MATRIX WHICH IS 'T2VV' IN DENST FUNC.
[3]  U←D[1]
[4]  V←D[2]
[5]  R←D[3]
[6]  P1←¯2×U×V×((1○K)*2)
[7]  P2←P1*2
[8]  P2←(P2+4×(U*2)×((((2○K)*2)×((U*2)+(V*2)))-V*2))
[9]  I←P2≥0
[10] P2←P2×I
[11] P2←P2*0.5
[12] P3←2×(U*2)
[13] J←1≥|((P1+P2)÷P3)
[14] TT1←(○(1-J))+(¯2○(J×((P1+P2)÷P3)))×J
[15] TT2←(○(1-J))+(¯2○(J×((P1-P2)÷P3)))×J
[16] TT1←(○(1-I))+I×TT1
[17] TT2←(○(1-I))+I×TT2
[18] T2D←TT1[10+⍳140;⍳210],TT2[10+⍳140;210+⍳190]
[19] T2UL←¯1×TT1[⍳10;⍳200]
[20] T2UR←TT1[⍳10;200+⍳10],TT2[⍳10;210+⍳190]
[21] T2U←T2UL,T2UR
[22] T2←T2U,[1] T2D
     ∇
```

Figure B.7 .

```
     ∇ T1 RHO T2;X;Y;XY;X1;Y1;X1Y1;ALFA;ALFAP
[1]  RH←1-(T1-T2)÷○2
[2]  RH←(⊖RH),[1] RH
[3]  X←⍳200
[4]  Y←⍳150
[5]  XY←(((X*2)∘.+(Y*2))*0.5)>10.1
[6]  XY←(⌽XY),XY
[7]  XY←⍉(⊖XY),[1] XY
[8]  X1←((⌽(⍳10)),¯1×⍳140)÷10
[9]  Y1←((⌽⍳200),¯1×⍳200)÷10
[10] ALFA←¯1○(U÷V)
[11] ALFAP←¯3○((R-X1)∘.÷((Y1+V)+1E¯8))
[12] X1Y1←ALFAP≤ALFA
[13] X1Y1←X1Y1[⍳150;⍳(200+V×10)],(150,(200-V×10))⍴0
[14] X1Y1←(⊖X1Y1),[1] X1Y1
[15] RH←RH×XY
[16] RH←(RH×X1Y1)+(1-X1Y1)
[17] RH[⍳60;110+⍳90]←1
     ∇
```

Figure B.8 .

APPENDIX C

SIMULATION

This simulation program is used to observe how the total area searched changes as a funcion of u, v and t.

```
//SEARCH JOB (4855,9999),'ROCKOWER ',CLASS=J
//  EXEC FORTVCG
//FORT.SYSIN DD *
C     SUBMARINE SEARCH AND DETECTION SIMULATION
C     PROF. E. ROCKOWER              WINTER 1987
C
       DIMENSION V0(7), U0(30000), NDET(30), XAXIS(0:30)
       REAL NSURV(0:30), DET(10)
       DATA V0/.25, .5, .75, 1.0, 2.0, 5.0, 10./
       U = 1
       R = 1
       TMAX = 30.
       YMAX = TMAX*U +R
       YMIN = -YMAX
       U0(1) = .50331
C   DO TEN REPLICATIONS OF THE SIMULATION
       DO 444 KK = 1,10
       ISEED = 337711*(U0(1) +.5)
       CALL LRND(ISEED, U0, 15000, 1, 0)
       TWPI = 2.*3.14159
       DO 100 I = 1, 7
       V = V0(I)
       K = 0
         DO 33 K = 1, 30
         NDET(K) = 0.0
 33     CONTINUE
       XMAX = (U+V )*TMAX + R
       XMIN = -R -  MAX (0., (U-V ) )*TMAX
            AREA = (YMAX-YMIN)*(XMAX-XMIN)
            NUMITS=IFIX(AREA/3.)
            NSURV(0)=NUMITS
       DO 200 NUMTGT = 1, NUMITS
       XS = 0.
 5    K = K + 1
       XT = XMIN + (XMAX - XMIN)*U0(3*K)
       YT = YMIN + (YMAX - YMIN)*U0(3*K-1)
       DISTO = XT**2 + YT**2
       IF ( DISTO .LE. R**2) GOTO 5
       THETA = TWPI*U0(3*K-2)
       UX = U*COS(THETA)
       UY = U*SIN(THETA)
       DT = R/10./(U + V)
       DO 300 T = 0., TMAX, DT
       XS = XS + V*DT
       XT = XT + UX*DT
       YT = YT + UY*DT
       DIST = (XT-XS)**2 + YT**2
         IF( DIST .LE. R**2)     GOTO 10
         IF((XT .GE. XMAX).OR. (XT .LE. XMIN) .OR. (YT .GE. YMAX)
      /    .OR. (YT. LE. YMIN) .OR. (DIST .GT. DISTO)) GOTO 200
       DISTO = DIST
 300  CONTINUE
       GOTO 200
 10   IT =  IFIX( T + .9999)
```

```
      NDET(IT) = NDET(IT) + 1
 200  CONTINUE
      DO 500 J = 1, 30
         NSURV(J) = NSURV(J-1) - NDET(J)
         XAXIS(J) = J
 500  CONTINUE
      XAXIS(0) = 0
      DET(I) =       NUMITS-NSURV(30)
      WRITE(6,*) I, DET(I)
 100  CONTINUE
      WRITE(6,*) ISEED
 444  CONTINUE
      STOP
      END
/*
//
```

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Thesis
C53135 Colak
c.1 A model evaluating the effectiveness of a systematic search for a moving target.